Bargaining Orders in a Multi-Person Bargaining Game[*]

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December 1, 2017

Abstract

This paper studies a complete-information bargaining game with one buyer and multiple sellers of different "sizes" or bargaining strengths. The bargaining order is determined endogenously. With a finite horizon, there is a unique subgame perfect equilibrium outcome, in which the buyer purchases in order of increasing size–from the smallest to the largest. With an infinite horizon, if the sellers have sufficiently different sizes, there is a unique equilibrium outcome, which has the same bargaining order. If the sellers have similar sizes with an infinite horizon, there may be multiple equilibrium outcomes with different bargaining orders.

JEL classification: C78, L23

Keywords: multi-person bargaining, bargaining order

1   Introduction

Consider a scenario in which a real estate developer must acquire land from multiple sellers. The sellers' lots are of different sizes with a larger lot giving a higher flow of payoffs to its owner. Such situations are quite common. For example, in Chongqing, China, the construction of a retail mall required 280 properties of different residents. The project was suspended for three years because one out of the 280 owners refused to sell his property to the developer.[1] Columbia University's expansion plan in West Manhattanville is another prominent example. The 17-acre project was worth 6.3 billion dollars, and the land was acquired from 67 separate property owners. The entire negotiation lasted over a long period from years 2002 to 2010, and

[1]The negotiation began in 2004, and eventually the owner sold his property in 2007. See "In China, Fight Over Development Creates a Star," *New York Times*, March 26, 2007, or "Nail House in Chongqing Demolished," *China Daily*, April 3, 2007.

the negotiation on the last three properties alone took more than three years.[2] What should the buyer (developer) do when she needs to purchase land from multiple sellers who own lots of different sizes? In particular, which seller should she bargain with first, the one with a large lot or a small lot? This paper examines the corresponding non-cooperative bargaining game. We find that the buyer should bargain with the seller of the smallest lot first, especially when the sizes of the lots are quite different. This paper does not try to explain the delay in the examples above. Delays occurred even when there was only one seller remaining in the first example, perhaps due to incomplete information.[3]

While the model studied here is couched in the language of a single developer negotiating with multiple sellers, it is applicable to a variety of other bargaining scenarios. For example, consider an airline that must bargain with two separate unions, pilots and flight attendants, in order to end a strike. Both unions are necessary for the airline to operate, but their outside options differ.[4] Which union should the firm negotiate with first? A similar question can be asked about the negotiation between a manufacturer and a group of upstream suppliers producing parts at different costs.[5] Our model also applies to the case in which a good, in order to reach the buyer, needs to pass a sequence of intermediaries with different transaction costs.[6] The key characteristics common to these scenarios are: the one-to-many aspect of the negotiation; the fact that an agreement with *all* sellers is necessary to reap any economic gains; and finally, the "size" differences among the sellers.

In this paper, bargaining strength is measured by the size of the inside/outside options available to a seller when bargaining with the buyer.[7] A seller with a large lot is stronger than a seller with a smaller lot in the sense that, in equilibrium, the price received by the large seller is higher than that received by the small seller. There are other notions of bargaining strength, of course. For instance, one may measure bargaining strength by how patient a seller is and different sellers may have different discount rates. Alternatively, it may be related to the likelihood of making initial offers (see, for instance, Li 2010).

It is useful to begin with a simple example. Consider a scenario with one developer and two farmers. All parties share a discount factor of $\delta \in (0, 1)$. Farmer 1 owns a large lot of land that produces $(1 - \delta)/2$ units of harvest in each period; farmer 2 owns a small lot of land that produces $(1 - \delta)/10$ units of harvest in each period. The land does not produce any harvest once it is sold to the developer. The developer must purchase both lots to build a mall that produces $1 - \delta$ units of profit in each period. Therefore, the present value of all harvests is

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[2]See "2 Gas Stations, and a Family's Resolve, Confront Columbia Expansion Plan," *New York Times,* September 20, 2008; "Reaction to Court's Rejection of Manhattanville Eminent Domain," *News,* December 4, 2009; and "The Clever Capitalism of Nicholas Sprayregen," *CU Columbia Spectator,* September 16, 2010.

[3]See, for instance, Admati and Perry (1987) for how incomplete information can lead to delay.

[4]An outside option is the payoff that a player receives if he leaves the negotiation.

[5]Bargaining between a manufacturer and its upstream suppliers is discussed, for example, in Blanchard and Kremer (1997), and Bedrey (2009).

[6]Manea (2017) discusses this example along with others in a study of a different topic on intermediation in networks.

[7]An inside option is the payoff received by a seller while negotiations are ongoing (see, for example, Muthoo 1999). The analysis in Sections 2 and 3 focus on inside options, but our qualitative results would not be affected if sellers had outside options instead. Section 4 discusses outside option in more details.

$v_1 = 1/2$ for farmer 1 and $v_2 = 1/10$ for farmer 2, and the present value of the total profit of the mall is 1.

Negotiations are sequential. In any period, the developer negotiates with only one farmer. The developer first offers a price, which the farmer may accept or reject. If the offer is accepted, the developer proceeds to negotiate with the other farmer in the next period (in a standard two-player alternating offer bargaining game). If the offer is rejected, the farmer makes a counter-offer in the next period, which the developer may accept or reject. If the developer accepts this offer, she proceeds to negotiate with the other farmer. If the developer rejects the offer, she picks a farmer, who could be the same one as in the previous period, and negotiates with him in the same fashion, and so on. Which farmer should the developer bargain with first?

Notice that the developer can pick any remaining farmer to negotiate with, there is no restriction on the choice of bargaining order. However, there is a unique subgame perfect equilibrium outcome, in which the developer purchases from farmer 2 first and then from farmer 1. The equilibrium prices are explained below. The payment to the first farmer is a sunk cost to the buyer. Therefore, after farmer 2 sells his land, the surplus is $1 - v_1$, which is the difference between the value of the mall and the value of farmer 1's harvests. Our result can be best illustrated with $\delta \to 1$.[8] If δ converges to 1, farmer 1 receives $\delta/(1 + \delta) = 1/2$ of the surplus as in the Rubinstein bargaining game.[9] This implies that a surplus of $\frac{1}{2}(1 - v_1)$ is paid to farmer 1, so he sells at a price of $v_1 + \frac{1}{2}(1 - v_1)$.[10] Excluding the price for farmer 1, the remaining value of mall is $\frac{1}{2}(1 - v_1)$. As a result, the total surplus for farmer 2 and the buyer is $\frac{1}{2}(1 - v_1) - v_2$, which is also the difference between the remaining value of the mall and the value of farmer 2's harvests. Similarly, farmer 2 and the buyer split this surplus equally as in the Rubinstein bargaining game. Therefore, a surplus of $\frac{1}{2}\left[\frac{1}{2}(1 - v_1) - v_2\right]$ is paid to farmer 2, and the buyer's payoff is $\frac{1}{2}\left[\frac{1}{2}(1 - v_1) - v_2\right] = \frac{3}{40}$. In contrast, if the buyer purchases from seller 1 first instead, she would receive a payoff of $\frac{1}{2}\left[\frac{1}{2}(1 - v_2) - v_1\right] = -\frac{1}{40}$.

Our model builds on the model of Cai (2000) by introducing endogenous bargaining order and asymmetric sellers. His model is the extreme case of our infinite-horizon game if the farmers do not receive harvest. The bargaining order is fixed and rotates among the sellers in his paper. He finds multiple stationary subgame perfect equilibrium outcomes, and that delay can occur in some of them. In contrast, the sellers are asymmetric and the bargaining order is endogenous in our game, resulting in a *unique* subgame perfect equilibrium outcome.

Several papers have the feature that the bargaining orders are endogenously determined, but they are determined in a restricted way. Perry and Reny (1993) allow each player to decide when to make an offer, which implicitly allows for different bargaining orders. Stole and Zwiebel (1996), Noe and Wang (2004) and Bedrey (2009) study bargaining orders in finite-horizon bargaining games. Chatterjee and Kim (2005) focus on the bargaining orders, in which the buyer does not switch to another seller before an agreement. The literature on agenda

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[8]A larger discount factor could stand for shorter periods. As δ becomes larger, the harvest within each period becomes smaller, while the present value of the harvests remains the same.

[9]See Rubinstein (1982).

[10]The equilibrium prices are calculated according to Step I in the proof of Proposition 4.

formation also discusses orders, but the orders have a different meaning: sequences of different issues or tasks.[11] Contrary to our paper, this literature suggests that the most important issue should be discussed first.[12]

Li (2010) also allows endogenous bargaining orders, but his paper is very different from ours. In his paper, a seller's bargaining strength is measured by his likelihood to make the first offer in each bargaining. He finds multiple equilibria and that any selling order can be sustained. In our paper, a seller's bargaining strength is measured by the size of his inside option, and the bargaining game has a unique equilibrium outcome. Krasteva and Yildirim (2012) also use the likelihood of making an offer to represent bargaining strength. They study bargaining orders in a two-period game, and find that, in the presence of uncertain payoffs for the sellers, the buyer may prefer different bargaining orders depending on the sellers' likelihood to make the first offer.

The studies on holdout–sellers delaying negotiations–in bargaining also assume complementary sellers, as in our setup.[13] Those studies focus on a different question from ours. Specifically, they focus on which exogenous bargaining features (e.g. transparency, sellers' complementarity) cause holdout. In contrasts, we focus on bargaining orders, which are endogenously determined. In our setup, if the sellers have similar sizes, delay may arise.[14]

The remainder of the paper is organized as follows. Section 2 studies a bargaining game with a finite horizon. Then, an infinite horizon is considered in Section 3, in which Section 3.1 studies the case if the sellers' sizes are sufficiently different, and Section 3.2 studies the case with similar sizes. Section 4 discusses extensions and applications.

2   Bargaining with a Finite Horizon

Our model is a non-cooperative and complete-information bargaining game. The game has $N+1$ players including one buyer, B, and a set of sellers, $\{1, 2, \cdots, N\}$ with $N \geq 2$. Each seller (he) has one lot of land, and the buyer (she) must purchase every lot in order to build a mall. In other words, the lots are perfect complements for the buyer.

While seller i's land is in his possession, he receives a constant flow of harvests, which is referred as to his inside option in the literature. The value of the harvest in each period is $v_i(1-\delta)$ and is received at the end of each period, where $\delta \in (0,1)$ is the discount factor.[15] Therefore, the present value of seller i's harvests in t periods is $v_i(1-\delta)[1+\delta+...+\delta^{t-1}] \equiv H_{i,t}$. If seller i never sells his land, the present value of his infinitely many periods' harvests is $H_{i,\infty} = v_i$. We assume $v_1 > v_2 > \cdots > v_N > 0$, which means seller i's land is of larger size than seller $(i+1)$'s if every unit area of land is equally productive. The mall produces a constant profit in

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[11]See, for example, Fershtman (1990), Winter (1997), Reinhard and Matthias (2001) and Flamini (2007).

[12]See, for example, Winter (1997) and Flamini (2007).

[13]See, for example, Mailath and Postlewaite (1990), Menezes and Pitchford (2004) and Chowdhury and Sengupta (2012).

[14]See Lemma 6.

[15]All players are assumed to have the same discount factor, and the consequences of heterogeneous discount factors are discussed in Section 4.

each period, and the present value of all the profits is normalized to 1.

Timing We first consider a bargaining game with a finite horizon of T periods with $T \geq N$, and then consider in Section 3 an infinite horizon. The bargaining game is a natural extension of the Rubinstein bargaining game.[16] More precisely, at the beginning of the game, the buyer picks one seller and bargains with him for one round. Each round can have one or two periods. In the first period, the buyer suggests a price to the seller, the seller then decides to either accept it or reject it. If the seller accepts, this round ends with only one period. Otherwise, the seller suggests another price in the second period, which the buyer accepts or rejects. If an agreement is reached, the buyer pays the seller the agreed price right away and the seller leaves the game permanently before the harvests of the period are realized. If the seller's suggestion is rejected, the buyer picks one of the remaining sellers, who may be the same or different to the previous one, and bargains with him in the next round in the same fashion. Note that the length of each round is endogenous and depends on the strategies. At the end of each period, every remaining seller receives a harvest from his land.

Note that there is no restriction on the bargaining order in the sense that the buyer can choose any remaining seller to bargain with. For example, before the first agreement, the buyer may always choose seller 1 to bargain with, or alway choose seller 2, or switch between the two sellers with any frequency. Figure 1 illustrates the two-seller bargaining game for $T = 3$, where $G(i, j, t)$ is the t-period two-seller game in which player i offers to j in the first period, and $G(i, t)$ is the t-period one-seller game in which the buyer offers to seller i in the first period. Moreover, $p_{i,t}^n$ denotes the buyer's offer to seller i in the n-seller game when there are t periods left, and $q_{i,t}^n$ denotes seller i's offer to the buyer in the n-seller game when there are t periods left. Throughout the paper, the superscripts, such as those in $p_{i,t}^n$ and $q_{i,t}^n$, denote the number of sellers in the game, but they are omitted if there is no ambiguity on the number of sellers.

Two features are important. First, the buyer bargains with only one seller at a time. This feature represents the case in which it is costly to communicate with all the sellers at the same time. In the example by Coase (1960), a railway company has to bargain with the farmers along a railway track. It is difficult to make simultaneous offers to multiple farmers when they are located far away from each other.[17] Second, the payments are made immediately after the corresponding agreements, which are referred as to cash-offer contracts. These contracts are prevalent in practice and in theoretic research of bargaining.[18] For example, electronics companies often need to purchase multiple patent licenses controlled by different owners in order to launch a product. Delays in payment may be unacceptable as negotiation could reveal some key ideas behind the patent. Additional examples and the consequences of relaxing these

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[16]However, our results are not specific to the setup with alternating offers. For instance, suppose that in each period, the buyer or the seller is selected with some probability to make an offer, which the other accepts or rejects. We can obtain the same equilibrium bargaining order. In addition, if the buyer and the seller chosen by her undertake Nash bargaining instead of making alternating offers, we can get the same results on bargaining order as well.

[17]This feature is also studied by Cai (2000, 2003) and Noe and Wang (2004).

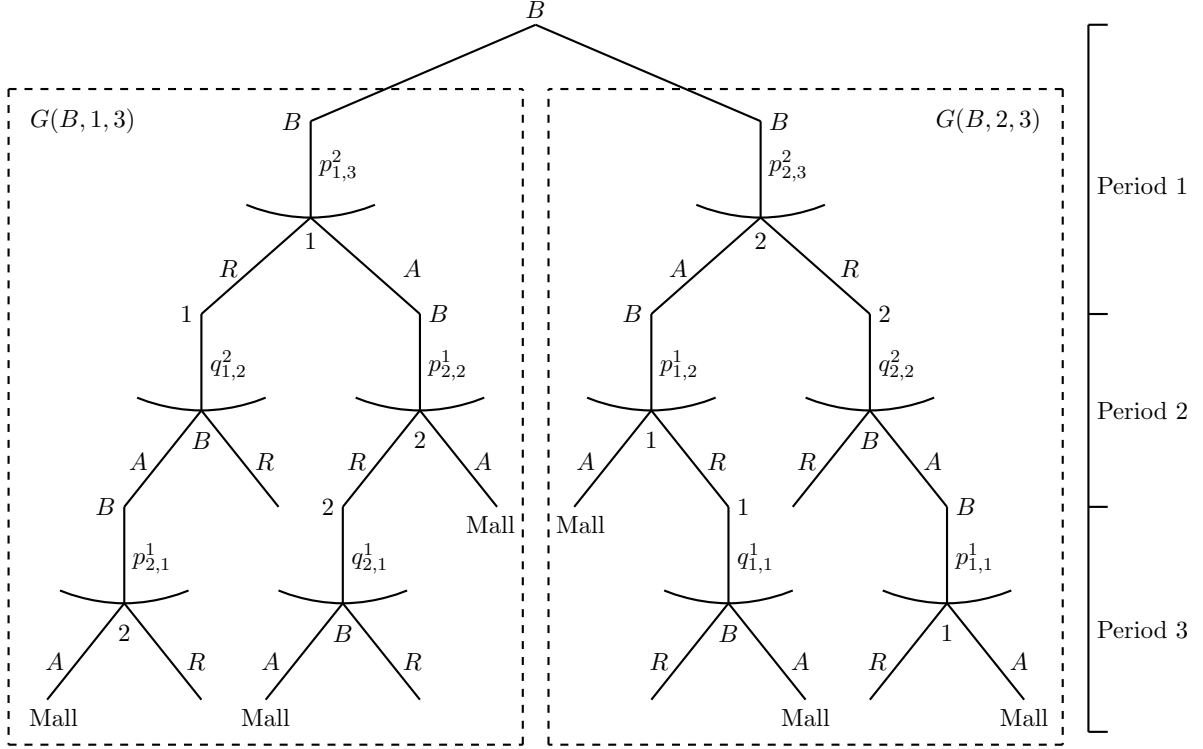[18]See, for instance, Krishna and Serrano (1996) and Cai (2000, 2003).

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Figure 1: Two-seller Game with Three Periods

two assumptions are discussed in Section 4.

Payoffs   Let t_i be the period in which seller i sells his land and p_i be the price. If seller i never sells his land, $t_i = \infty$. Given t_i and p_i, seller i's payoff is

$$\pi_i = H_{i,t_i-1} + \delta^{t_i-1}p_i \tag{1}$$

where the first term is the present value of his harvests prior to the sale of his land, and the second term is the present value of the buyer's payment to the seller.

The buyer cannot reap the harvests from the land, which represents the scenarios in which the buyer cannot fully utilize the land (as the sellers could) before the mall is built. Take the land purchasing case in Chongqing as an example. The sellers received utility by living in their houses, but the buyer could not receive the same amount of total utility even if she owns the houses. As a result, the buyer's payoff is

$$\pi_B = \delta^{\max(t_1,\cdots,t_N)-1} - \sum_{i=1}^{N} \delta^{t_i-1}p_i \tag{2}$$

where the first term is the present value of the mall and the second term is the present value of the payments to the sellers. Seller i's surplus is $\pi_i - v_i$, which is his payoff minus the present value of his harvests. The buyer's surplus is simply her payoff because she receives no harvest.

In this paper, we only consider subgame perfect Nash equilibria. Given an equilibrium, its

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outcome is denoted as $(p_1, p_2, ..., p_N, t_1, t_2, ..., t_N)$. We assume that if a player would receive a zero surplus in every equilibrium in the bargaining game, the player chooses not to participate the bargaining game. This assumption eliminates equilibria in which a player rejects for many periods and then accepts an offer that gives him/her a zero surplus.[19] Such equilibria are not robust to an arbitrarily small cost of bargaining. The following result establishes the unique equilibrium outcome.

Proposition 1 *For any $N \geq 2$ and any $T \geq 2$, the N-seller game with horizon T has a unique equilibrium outcome. Moreover, if the mall is built in the outcome, then in the first N periods the buyer purchases from the N sellers in the order of increasing size.*

Due to its length, the proof of Proposition 1 is omitted here and can be found in an online appendix. The proof is based on backward induction. For a small number of sellers (e.g. $N = 2$) and a short horizon (e.g. $T = 2$ or 3), it is easy to verify the proposition. However, for more players and longer horizons, the analysis becomes much more involved because the equilibrium outcome and the condition for the mall to be built vary with horizon T and depend on whether T is even or odd.[20] In contrast, as the horizon goes to infinity, the limiting equilibrium outcome and the condition for the mall to be built are simpler. Proposition 2 characterizes the equilibrium outcome and the condition in the limit. The following condition is referred to as the profitability condition and is used repeatedly in the remainder of the paper:

$$\sum_{i=1}^{N} \left(\left(\frac{\delta}{1+\delta} \right)^{N-i} v_i \right) < \left(\frac{\delta}{1+\delta} \right)^{N-1} \tag{3}$$

which requires that the land's sizes, measured in $v_1, ..., v_N$, are not too big. An intuition for this condition is that when the buyer bargains with seller N in period 1, she expects the other sellers to sell in the following $N - 1$ periods. In addition, if the mall is eventually built, then in period 1 the buyer should expect a positive total surplus for seller N and herself given the prices to be paid to the other sellers. Condition (3) ensures such a positive total surplus if the horizon goes to infinity. In an N-seller game with horizon T, let $p_{i,T}^N$ denote seller i's selling price and $\pi_{B,T}^N$ denote the buyer's payoff.

Proposition 2 *Suppose (3) holds, then the mall is built in the N-seller game if the horizon is long enough. Moreover, the smallest seller, N, sells in period 1 and his price satisfies*

$$\lim_{T \to \infty} p_{N,T}^N = v_N + \frac{\delta}{1+\delta} \left[\delta \lim_{T \to \infty} \pi_{B,T}^{N-1} - v_N \right]$$

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[19]Without this assumption, if the horizon is $2t > 4$ in Example 1, there is no agreement until only two periods remain, when seller 2 accepts a price of v_2 and seller 1 accepts a price of v_1. Both sellers' surplus is zero.

[20]The parity of T closely relates to whether a seller or the buyer makes the last offer. See the online appendix for the characterization of the condition and equilibrium outcome.

and the buyer's surplus satisfies

$$\lim_{T\to\infty} \pi_{B,T}^N = \frac{1}{1+\delta}\left[\delta \lim_{T\to\infty} \pi_{B,T}^{N-1} - v_N\right]$$

If (3) does not hold, the mall is not built for any horizon.

The proof uses Proposition 1 and its proof as a building block, and it is in the appendix. Notice that after the first purchase, the subsequent subgame is an $(N-1)$-seller game. Therefore, using Proposition 2, we can derive the limiting price for each seller recursively. The corollary below characterizes each seller's limiting price in the two-seller game.

Corollary 1 *In the two-seller game, if horizon T is long enough and if $\frac{\delta}{1+\delta}(1-v_1) - v_2 > 0$, seller 2 sells in period 1 and his price satisfies*

$$\lim_{T\to\infty} p_{2,T}^2 = v_2 + \frac{\delta}{1+\delta}\left[\frac{\delta}{1+\delta}(1-v_1) - v_2\right] \tag{4}$$

seller 1 sells in period 2 and his price satisfies

$$\lim_{T\to\infty} p_{1,T-1}^1 = v_1 + \frac{\delta}{1+\delta}(1-v_1) \tag{5}$$

and the buyer's payoff satisfies

$$\lim_{T\to\infty} \pi_{B,T}^2 = \frac{1}{1+\delta}\left[\frac{1}{1+\delta}\delta(1-v_1) - v_2\right] \tag{6}$$

If $\frac{\delta}{1+\delta}(1-v_1) - v_2 \leq 0$, the mall is not built for any finite horizon.

The proof is in the appendix. Let us explain the intuition. After the first purchase, if the mall is built, its value is 1. In contrast, if the mall is not built, the present value of seller 1's harvests is v_1. Therefore, the surplus is the difference between these values, $1 - v_1$. The buyer and seller 1 split this surplus as if they are in the Rubinstein bargaining game. More precisely, the surplus paid to seller 1 is $\frac{\delta}{1+\delta}(1-v_1)$ according to (5). Therefore, with the surplus for seller 1 excluded, the mall would be worth $\frac{1}{1+\delta}(1-v_1)$ in period 2, or $\frac{\delta}{1+\delta}(1-v_1)$ in period 1. As a result, the agreement with seller 2 produces a surplus of $\frac{\delta}{1+\delta}(1-v_1)-v_2$, which is the remaining value of the mall minus the value of seller 2's harvests. It is easy to see from (4) and (6) that the buyer and seller 2 also split the surplus $\frac{\delta}{1+\delta}(1-v_1) - v_2$ as in the Rubinstein bargaining game, where (3) for $N = 2$ guarantees that this surplus is positive.

Let us explain why the buyer prefers to purchase from the smaller seller first. Suppose the buyer purchases from the larger seller first. Then, by the argument above, the buyer would receive a payoff of $\frac{1}{1+\delta}[\frac{\delta}{1+\delta}(1-v_2) - v_1]$, which is lower than that if she purchases from the smaller seller first.

In our model explicated above, the heterogeneity in the sellers' inside options allows the selling orders to affect the surplus after the first purchase. However, this may not be true for

other types of heterogeneity, and there could be multiple equilibrium outcomes. For example, Li (2010) considers the heterogeneity in sellers' probabilities to make the first offer, so no matter who sells first, the surplus after the first purchase is the same. He finds multiple equilibrium outcomes with different bargaining orders.

It is also worth mentioning that there may be multiple equilibria associated with the unique outcome, which is illustrated in the following example.

Example 1 *Consider a two-seller game with $T = 4$, $v_1 = 0.3$, $v_2 = 0.1$ and $\delta = 0.8$.*

By backward induction, we can verify that there are at least two equilibria, which have different strategies off the equilibrium path. In one equilibrium, in the subgame given every player's participation, the buyer bargains with seller 2 first. No agreement is reached in the first two periods. Then, seller 2 sells first in period 3 at a price of v_2, and seller 1 sells in period 4 at a price of v_1. To see why there is delay of two periods, suppose that seller 2 sells in period 2. His price should not be lower than v_2. After seller 2 sells, seller 1 sells in period 3 at a price of $v_1 + \delta(1 - v_1) = 0.86$.[21] Therefore, the buyer's payoff evaluated in period 2 is no more than $\delta(1 - 0.86) - v_2 = 0.012$. However, the buyer could wait one period and get a payoff of $\delta(1 - v_1) - v_2 = 0.46$, so her payoff in period 2 should be at least $0.46\delta = 0.368$, which exceeds 0.012. This is a contradiction. Similarly, we can show that no seller would sell in the first or second period.

In another equilibrium, the buyer bargains with seller 1 first. No agreement is reached in the first two periods. Then, sellers 2 and 1 sell in periods 3 and 4 respectively as in the first equilibrium. Each seller receives a surplus of zero in both equilibria, so both sellers do not participate the bargaining game. Thus, the equilibria have the same outcome in which the mall is not built. However, since no agreement can be reached in the first two periods of the bargaining game, the buyer may bargain with either seller first in equilibrium.

3   Bargaining with an Infinite Horizon

In this section, we consider the same bargaining game but with an infinite horizon. Figure 2 illustrates the game tree for the two-seller game, where $\Gamma(i)$ is the one-seller game between seller i and the buyer who offers in period 1, and $\Gamma(B, j)$ is the two-seller game in which the buyer offers to seller j in period 1. Next, we discuss two cases. First, Section 3.1 discusses sellers of sufficiently different sizes, and shows a unique equilibrium outcome. Second, Section 3.2 discusses sellers of similar sizes, and obtains multiple equilibrium outcomes.

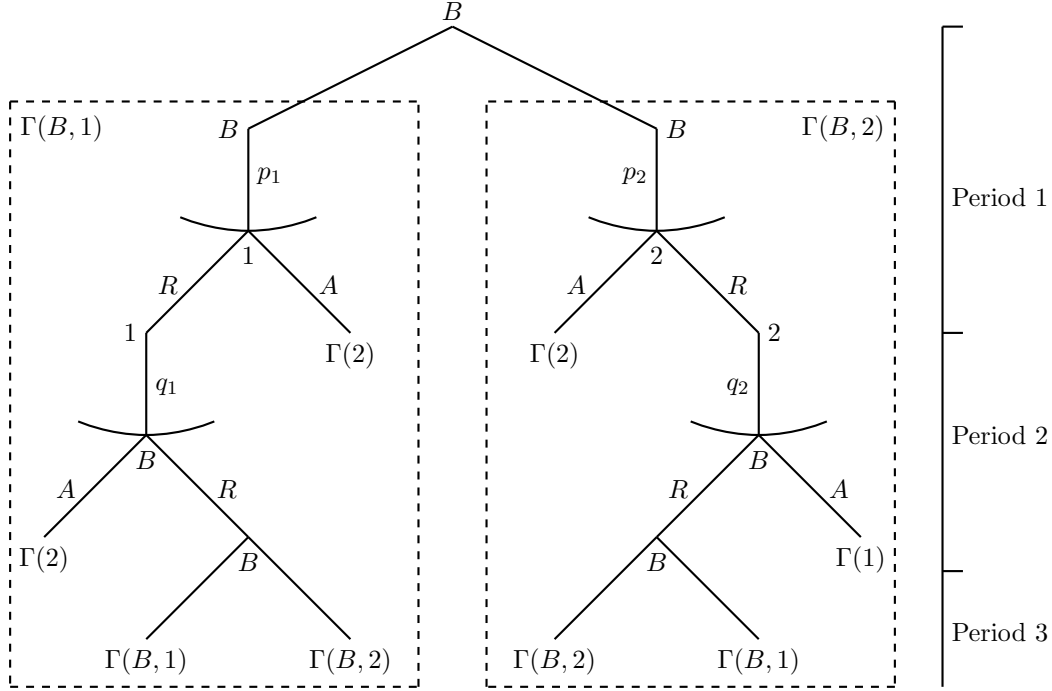[21]The calculation is based on Claims 1 and 2 in the online appendix.

Figure 2: Two-Seller Game with an Infinite Horizon

3.1 Sellers with Sufficiently Different Sizes

The following condition is repeatedly used in the analysis below:

$$\left(1 - \frac{\delta}{\delta + 1}\right)(v_{n-1} - v_n) + \sum_{i=1}^{n-1}\left((v_i - v_{i+1})\sum_{j=n-i}^{n-1}\left(\frac{\delta}{1+\delta}\right)^j\right)$$
$$> \left(\frac{\delta}{\delta+1}\right)^{n-1} - v_n \sum_{i=1}^{n}\left(\frac{\delta}{1+\delta}\right)^{i-1} \tag{7}$$

which requires that in an n-seller game, the differences in sellers' sizes, measured in $v_i - v_{i+1}$ for $i = 1, ..., n-1$, are not too small. Under this condition, the result below establishes a unique equilibrium outcome.

Proposition 3 *a) If the mall is profitable as in (3) and if the sellers' sizes are sufficiently different as in (7) for $n = 2, ..., N$, then the N-seller game with an infinite horizon has a unique equilibrium outcome, and the buyer purchases from the N sellers in the first N periods in the order of increasing size.*

b) If (3) holds but (7) does not hold for all $n = 2, ..., N$, it remains an equilibrium outcome that the buyer purchases from the N sellers in the first N periods in the order of increasing size.

c) If (3) does not hold, the mall is not built.

In part a), the mall is built without delay in the sense that the N sellers sell in the first N periods.[22] Condition (3) is referred as to the profitability condition because the buyer's

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[22]See Lemma 3 for the equilibrium strategies.

equilibrium payoff is positive if and only if (3) holds. To see this, notice that if (3) holds, the buyer's equilibrium payoff must be positive, otherwise she would not participate and the mall would not be built. If (3) does not hold, the mall is not built, so the buyer's payoff is zero. Condition (7) ensures that the sellers' sizes are different enough so that when the remaining sellers are $1, 2, ..., n$, seller n sells first. To see why, recall that in the introduction, we illustrate in an example that if a larger seller sells before a smaller one, the buyer would receive a lower payoff. Moreover, the lower payoff could be negative if the sellers have significantly different sizes. Condition (7) ensures that the size differences are large enough so that if seller n did not sell before the larger ones, $1, ..., n - 1$, the buyer would receive a negative payoff. If (7) is violated for some n, different bargaining orders may arise in equilibria, which are discussed in Section 3.2.

In the remainder of Section 3.1, we prove Proposition 4, which is a special case of Proposition 3 in a two-seller game, to illustrate the main idea. The proof of Proposition 3 for the N-seller game is in the appendix.

First, we consider the subgame after the first purchase. Because the payment to the first seller is a sunk cost to the buyer, the subgame after the first purchase is a one-seller game between the buyer and the remaining seller. Moreover, this one-seller game is simply a two-person alternating bargaining game with inside options only available to the seller. Define

$$
\begin{aligned}
p_i^1 &= v_i + \frac{\delta}{1+\delta}(1 - v_i) \\
q_i^1 &= 1 - \frac{\delta}{1+\delta}(1 - v_i)
\end{aligned}
\tag{8}
$$

The following result characterizes the unique equilibrium in the one-seller game.

Lemma 1 *In the infinite-horizon one-seller game between the buyer and seller i, there is a unique equilibrium. In the equilibrium,*
i) the seller offers a price of q_i^1 and accepts a price no less than p_i^1,
ii) the buyer offers a price of p_i^1 and accepts a price no more than q_i^1.

The proof of Lemma 1 is a straightforward adaptation of Proposition 6.1 of Muthoo (1999), so we omit it here. Similar to Muthoo's result, each player is indifferent between accepting and rejecting the other player's offer in the equilibrium. Indeed, we can verify that

$$
\begin{aligned}
p_i^1 &= H_{i,1} + \delta q_i^1 \\
1 - q_i^1 &= \delta(1 - p_i^1)
\end{aligned}
\tag{9}
$$
$$
\tag{10}
$$

where (9) means that the seller is indifferent between accepting and rejecting the buyer's offer p_i^1, and (10) means that the buyer is indifferent between accepting and rejecting the seller's offer q_i^1. Moreover, the equilibrium payoffs are $v_i + \frac{\delta}{1+\delta}(1 - v_i)$ for the seller and $\frac{1}{1+\delta}(1 - v_i)$ for the buyer, so the seller receives a fraction $\frac{\delta}{1+\delta}$ of the total surplus $1 - v_i$ and the buyer receives $\frac{1}{1+\delta}$

of it. Note that they split the total surplus in the same way as in the Rubinstein bargaining game. Next, we consider a two-seller game, in which Proposition 3 reduces to:

Proposition 4 *a) In the infinite-horizon two-seller game, if the mall is profitable as in (3) with $N = 2$ and if the sellers' sizes are sufficiently different as in (7) with $n = 2$, there is a unique equilibrium outcome, in which the buyer purchases from seller 2 in period 1 and from seller 1 in period 2.*

b) If (3) holds for $N = 2$ but (7) does not hold for $n = 2$, it remains an equilibrium outcome that the buyer purchases from seller 2 in period 1 and from seller 1 in period 2.

c) If (3) does not hold for for $N = 2$, the mall is not built.

Proof. With $N = 2$, condition (3) reduces to

$$\frac{\delta}{1+\delta}(1 - v_1) - v_2 > 0 \tag{11}$$

and with $n = 2$, condition (7) reduces to

$$\frac{\delta}{1+\delta}(1 - v_2) - v_1 < 0 \tag{12}$$

Lemma 1 already characterizes the unique equilibrium in the subgame after the first purchase, so we only need to discuss the strategies *before* the first purchase. In the remainder of the proof, Step I characterizes a set of equilibria, Step II shows a unique equilibrium outcome and proves part a), and Step III verifies parts b) and c).

Step I. Define

$$p_2^2 = v_2 + \frac{\delta}{1+\delta}[\delta(1 - p_1^1) - v_2] \tag{13}$$

$$q_2^2 = \delta\left(1 - p_1^1\right) - \frac{\delta}{1+\delta}\left[\delta\left(1 - p_1^1\right) - v_2\right] \tag{14}$$

$$p_1^2 = H_{1,3} + \delta^3 p_1^1 \tag{15}$$

$$q_1^2 = \delta(1 - p_2^1) - \delta(\delta(1 - p_1^1) - p_2^2) \tag{16}$$

We claim that for any (q_{1B}^2, p_{B1}^2) such that $q_{1B}^2 > q_1^2$ and $p_{B1}^2 < p_1^2$, the strategies below constitute an equilibrium:

i) seller 1 suggests a price of q_{1B}^2 and accepts a price no less than p_1^2,

ii) seller 2 suggests a price of q_2^2 and accepts a price no less than p_2^2,

iii) the buyer bargains with seller 2 until an agreement is reached; suggests a price p_2^2 to seller 2 and p_{B1}^2 to seller 1; accepts a price no more than q_2^2 from seller 2 and no more than q_1^2 from seller 1.

To verify the above strategies indeed constitute an equilibrium, we first calculate the resulting payoffs from the strategies. According to the strategies, seller 2 sells at price p_2^2 in period

1. Substituting p_2^2 into (1), we obtain seller 2's payoff

$$\pi_2^* \;\;=\;\; v_2 + \frac{\delta}{1+\delta}\left[\delta\frac{1}{1+\delta}(1-v_1)-v_2\right] \tag{17}$$

After seller 2 sells, Lemma 1 shows that seller 1 sells at price p_1^1 in period 2, so his payoff is

$$\pi_1^* = v_1 + \delta\frac{\delta}{1+\delta}(1-v_1) \tag{18}$$

Substituting p_2^2 and p_1^1 into (2), we obtain the buyer's payoff

$$\pi_B^* \;\;=\;\; \frac{1}{1+\delta}\left[\delta\frac{1}{1+\delta}(1-v_1)-v_2\right] \tag{19}$$

Notice that the buyer and seller 2's surpluses π_B^* and $\pi_2^* - v_2$ are positive due to (11), and seller 1's surplus $\pi_1^* - v_1$ is positive due to $v_1 < 1$.

Next, we verify that no player benefits from deviating. We can verify that q_1^2 satisfies

$$\delta(1-p_2^1) - q_1^2 = \delta(\delta(1-p_1^1)-p_2^2) \tag{20}$$

where the left hand side is the buyer's payoff if she accepts price q_1^2 from seller 1, and the right hand side is $\delta\pi_B^*$. Therefore, (20) implies that the buyer is indifferent between accepting q_1^2 and rejecting it. As a result, the buyer accepts prices no higher than q_1^2 from seller 1. However, (12) implies $q_1^2 < v_1$, so seller 1 cannot afford any price that the buyer accepts. Therefore, seller 1 suggests any price q_{1B}^2 above the buyer's threshold of acceptance, q_1^2, and does not deviate.

The left hand side of (15) is seller 1's payoff if he accepts p_1^2. The right hand side of (15) is π_1^* with two periods of delay, which is seller 1's payoff if he rejects p_1^2. Hence, the equation implies that seller 1 is indifferent between accepting p_1^2 and rejecting it. As a result, seller 1 accepts a price no less than p_1^2. However, if the buyer offers any price higher than p_1^2, her payoff is lower than $\frac{\delta}{1+\delta}(1-v_2)-p_1^2 \leq \frac{\delta}{1+\delta}(1-v_2)-v_1 < 0$, where the first inequality is from $p_1^2 \geq v_1$ and the second from (12). Therefore, the buyer offers p_{B1}^2 below seller 1's threshold of acceptance, p_1^2, and does not deviate.

We can rewrite (13) and (14) as

$$p_2^2 \;\;=\;\; H_{2,1} + \delta q_2^2 \tag{21}$$
$$\delta\left(1-p_1^1\right) - q_2^2 \;\;=\;\; \delta(\delta(1-p_1^1)-p_2^2) \tag{22}$$

According to (22), the buyer is indifferent between accepting q_2^2 from seller 2 and rejecting it. Hence, the buyer would not change her threshold of acceptance, q_2^2, and seller 2 would not change his offer q_2^2. According to (21), seller 2 is indifferent between accepting p_2^2 from the buyer and rejecting it. Hence, the seller would not deviate from her threshold of acceptance, p_2^2, and the buyer would not deviate from his offer p_2^2. Hence, there is no deviation and we prove the claim in Step I.

Step II. We verify the uniqueness of equilibrium outcome. To prove this, we first show in Lemma 2 that perpetual disagreement is not an equilibrium outcome. The proof is in the appendix. As a result, the first seller sells after a finite number of periods. We claim that the first seller must be seller 2. To see this, suppose seller 1 sells first in period t. Then, seller 2 sells in period $t + 1$. Similar to (19), the buyer receives a payoff of

$$\delta^{t-1} \frac{1}{1+\delta} \left[\delta \frac{1}{1+\delta} (1 - v_2) - v_1 \right]$$

which is negative due to (12). Therefore, the first seller must be seller 2.

As a result, whenever seller 2 and the buyer agree upon a price, the total surplus of these two players in every equilibrium is $\delta(1 - p_1^1) - v_2 \equiv S_{-1}$, which is the total surplus $\delta(1 - v_1) - v_2$ minus seller 1's surplus received in the following period $\delta(p_1^1 - v_1)$. Hence, the bargaining game reduces to a bilateral bargaining game between seller 2 and the buyer to split a total surplus of S_{-1}. Thus, we can use the method for two-player bargaining games to show the uniqueness of equilibrium outcome. In particular, the rest of Step II follows closely Section 4.4.2 of Fudenberg and Tirole (1991).[23]

Recall that the buyer's surplus is her payoff, while a seller's surplus is his price minus his value. Now we define m_B and M_B as the infimum and supremum of the buyer's equilibrium surpluses in subgame $\Gamma(B, 2)$, in which the buyer offers to seller 2 in period 1.[24] Let w_2 and W_2 be the infimum and supremum of seller 2's equilibrium surpluses in subgame $\Gamma(B, 2)$. Similarly, let m_2 and M_2 be the infimum and supremum of equilibrium seller 2's surpluses in subgame $\Gamma(2, B)$, and w_B and W_B be the infimum and supremum of the buyer's equilibrium surpluses in subgame $\Gamma(2, B)$.

Next, we prove the uniqueness of equilibrium outcome. In $\Gamma(B, 2)$, seller 2 will accept any price such that his surplus exceeds δM_2, since he cannot expect more than M_2 in $\Gamma(2, B)$ following his refusal. Hence, $m_B \geq S_{-1} - \delta M_2$. By the symmetric argument, the buyer accepts any price such that her surplus exceeds δM_B. Hence, $m_2 \geq S_{-1} - \delta M_B$.

Since seller 2 will never suggest a price such that the buyer's surplus exceeds δM_B, the buyer's surplus W_B in $\Gamma(2, B)$ is at most δM_B. That is, $W_B \leq \delta M_B$.

Since seller 2 can obtain a surplus of at least m_2 in $\Gamma(2, B)$ by rejecting in $\Gamma(B, 2)$. Seller 2 will reject any price such that his surplus is below δm_2. Therefore, the buyer's highest equilibrium surplus in $\Gamma(B, 2)$, M_B, satisfies $M_B \leq \max(S_{-1} - \delta m_2, \delta W_B) \leq \max(S_{-1} - \delta m_2, \delta^2 M_B)$. Next, we claim that $\max(S_{-1} - \delta m_2, \delta^2 M_B) = S_{-1} - \delta m_2$. If not, then we would have $M_B \leq \delta^2 M_B$, implying $M_B \leq 0$. Then, because m_2 cannot exceed S_{-1}, we have $S_{-1} - \delta m_2 > M_B$, contradicting the initial assumption $\max(S_{-1} - \delta m_2, \delta^2 M_B) \neq S_{-1} - \delta m_2$. Thus, $M_B \leq S_{-1} - \delta m_2$. By symmetry, $M_2 \leq S_{-1} - \delta m_B$. Combining these inequalities, we have $m_B \geq S_{-1} - \delta M_2 \geq S_{-1} - \delta(S_{-1} - \delta m_B)$ or $m_B \geq S_{-1}/(1 + \delta)$, and $M_B \leq S_{-1} - \delta m_2 \leq S_{-1} - \delta(S_{-1} - \delta M_B)$ or $M_B \leq S_{-1}/(1 + \delta)$; because $m_B \leq M_B$, this implies $m_B = M_B = S_{-1}/(1 + \delta)$. Similarly,

---

[23]Specifically, m_i, M_i, w_i and W_i for $i = B, 2$ in this paper correspond to their $\underline{v}_i$, $\bar{v}_i$, $\underline{w}_i$ and $\bar{w}_i$ for $i = 1, 2$.

[24]The infimum and supremum exist because of the existence of equilibria established in Step I.

$m_2 = M_2 = S_{-1}/(1+\delta)$, $w_B = W_B = S_{-1}\delta/(1+\delta)$, and $w_2 = W_2 = S_{-1}\delta/(1+\delta)$.

This shows that the equilibrium surpluses of the buyer and seller 2 are unique. Recall that the buyer's payoff is her surplus while a seller's surplus is his payoff minus his value, so their equilibrium payoffs are also unique and are given in (17) and (19) respectively. Notice that seller 2 and the buyer's total equilibrium payoff is $\delta(1 - p_1^1) - v_2$ with p_1^1 defined in (8), which means that seller 1, who is the second seller, sells in period 2 at price p_1^1. Therefore, seller 2 sells in period 1, and his equilibrium payoff in (17) implies that his selling price is p_2^2 described in (13). Hence, the equilibrium outcome must be $(p_1^1, p_2^2, 2, 1)$.

Step III. We prove parts b) and c). For part b), we start with the case in which

$$\delta(1 - p_2^1) - v_1 < \delta^2[\delta(1 - v_1) - p_2^2] \tag{23}$$

Consider the same strategies described in Step I. In Step I we use (12) to show that there is a two-period delay in $\Gamma(B, 1)$. This delay remains under condition (23). To see why, notice that in $\Gamma(1, B)$, the buyer is indifferent between accepting and rejecting q_1^2. Moreover, seller 1 offers q_1^2 if

$$q_1^2 - v_1 \geq \delta^2(p_1^1 - v_1) \tag{24}$$

which means by offering q_1^2, which the buyer accepts, seller 1's surplus is no lower than that if there is no agreement in period 1. Substituting q_1^2 described in (16) into (24), we can rewrite it as $\delta(1 - p_2^1) - v_1 \geq \delta[\delta(1 - v_1) - p_2^2]$, which is violated under condition (23). Thus, there is no agreement in period 1 of $\Gamma(1, B)$.

Given no agreement in period 1 of $\Gamma(1, B)$, seller 1 in $\Gamma(B, 1)$ is indifferent between accepting and rejecting p_1^2. Moreover, the buyer offers p_1^2 if

$$\delta(1 - p_2^1) - p_1^2 \geq \delta^2 \pi_B^* \tag{25}$$

which means by offering p_1^2 the buyer's payoff is no lower than that if she waits two periods and receives π_B^*. Substituting p_1^2 into the above inequality, we obtain $\delta(1-p_2^1)-v_1 \geq \delta^2(\delta(1-v_1)-p_2^2)$, which is also violated under condition (23). Thus, there is no agreement in periods 1 and 2 of $\Gamma(B, 1)$. Then, by the same argument in Step I, no player would deviate from the strategies described in the step. Hence, the outcome described in part a) remains an equilibrium outcome under condition (23).

Next, we show that without (23), the above equilibrium outcome remains. Consider the case in which

$$\delta^2[\delta(1 - v_1) - p_2^2] \leq \delta(1 - p_2^1) - v_1 < \delta[\delta(1 - v_1) - p_2^2] \tag{26}$$

In this case, (24) remains being violated but (25) becomes to hold. Therefore, the buyer offers p_1^2 in period 1 of $\Gamma(B,1)$ and seller 1 accepts it. The resulting payoff for the buyer is

$$
\begin{aligned}
\delta(1 - p_2^1) - p_1^2 &= \delta(1 - p_2^1) - v_1 - \delta^3(p_1^1 - v_1) \\
&= \delta(1 - p_2^1) - v_1 - \delta^2[\delta(1 - v_1) - p_2^2] + \delta^2\pi_B^* \\
&< \delta(1 - p_2^1) - v_1 - \delta[\delta(1 - p_2^1) - v_1] + \delta^2\pi_B^* \\
&< \delta(1 - p_1^1) - v_2 - \delta[\delta(1 - p_1^1) - v_2] + \delta^2\pi_B^* \\
&= \pi_B^*
\end{aligned}
$$

where the first equality is from (15), the second from $\pi_B^* = \delta(1 - p_1^1) - p_2^2$, the last from (19) and the first inequality from (26). Thus, the buyer chooses to bargain with seller 2 first. Then, following the analysis for $\Gamma(B,2)$ in Step I, seller 2 sells in period 1 at price p_2^2 and seller 1 sells in period 2 at price p_1^1. Consider another case in which

$$
\delta(1 - p_2^1) - v_1 \geq \delta[\delta(1 - v_1) - p_2^2] \tag{27}
$$

In this case, (24) holds, which means in $\Gamma(1,B)$ seller 1 offers q_1^2 and the buyer accepts it. Moreover, in $\Gamma(B,1)$, seller 1 is indifferent between accepting and rejecting a price of $v_1 + \delta(q_1^2 - v_1)$. In addition, the buyer offers this price if $\delta(1 - p_2^1) - [v_1 + \delta(q_1^2 - v_1)] \geq \delta^2\pi_B^*$. Substituting q_1^2 and π_B^* into this inequality, we obtain $\delta(1 - p_2^1) - v_1 \geq \delta[\delta(1 - p_2^1) - v_1]$, which is always true. Thus, in the case with (27), there is an agreement in period 1 of $\Gamma(B,1)$ and the buyer's payoff is $\delta(1 - p_2^1) - [v_1 + \delta(q_1^2 - v_1)] = (1 - \delta)[\delta(1 - p_2^1) - v_1] + \delta^2\pi_B^* < (1 - \delta)[\delta(1 - p_1^1) - v_2] + \delta^2\pi_B^* = \pi_B^*$, where the last equality is from (19). Therefore, the buyer also bargains with seller 2 first. Following the analysis for $\Gamma(B,2)$ in Step I, seller 2 sells in period 1 at price p_2^2 and seller 1 sells in period 2 at price p_1^1, which completes the proof of part b).

Next, we prove part c). Suppose the mall is built if (3) does not hold for $N = 2$. Then, as discussed in Step II, seller 2 must sell first. Suppose he sells in period t, then Lemma 1 implies that seller 1 sells in period $t + 1$. Then, the total surplus for the buyer and seller 2 is S_{-1} in period t. Substituting p_1^1 given in (8) into the definition of S_{-1}, we obtain $S_{-1} = \frac{\delta}{1+\delta}(1 - v_1) - v_2$, which is nonpositive if (3) does not hold for $N = 2$. This means the buyer and seller 2's total surplus is nonpositive. Hence, they choose not to participate the bargaining and the mall cannot be built. This is a contradiction. ∎

We discuss below the main idea to prove the unique equilibrium outcome. Seller 2 always sells first because the buyer would receive a negative payoff if seller 1 sold first. Then, after seller 2's sale, everything in the resulting subgame is known according to the unique equilibrium of the Rubinstein bargaining game. As a result, the multilateral bargaining game reduces to a bilateral bargaining game between the buyer and seller 2. Thus, the rest of the proof is parallel to the proof of the unique equilibrium outcome in the Rubinstein bargaining game.

Let us explain the implications of Proposition 4. Step I in its proof characterizes all the equilibria whose outcomes are $(p_1^1, p_2^2, 2, 1)$. Because the equilibrium outcome is unique according to Proposition 4, all the strategies are uniquely determined by backward induction except for q_{1B}^2 and p_{B1}^2, the rejected offers. As a result, Step I also describes all the equilibria of $\Gamma(B, 2)$.

Next, we discuss the bargaining orders in the equilibria. If seller 1 sells first, the buyer and seller 1 split a surplus of $\frac{\delta}{1+\delta}(1 - v_2) - v_1$, which is negative due to (12). Therefore, seller 1 does not sell first. If the buyer deviates to bargain with seller 1 first, the resulting subgame is $\Gamma(B, 1)$, which is a proper subgame of $\Gamma(B, 2)$. Therefore, the equilibria and the equilibrium outcome are inherited from Step I. Specifically, if there is no agreement in period 1 or 2, then the buyer chooses seller 2 to bargain with and seller 2 sells in period 3 and seller 1 sells in period 4. As a result, there is delay of two periods and the payoffs are $\pi_1^{*\prime} = H_{1,2} + \delta^2 \pi_1^*$, $\pi_2^{*\prime} = H_{2,2} + \delta^2 \pi_2^*$ and $\pi_B^{*\prime} = \delta^2 \pi_B^*$. This means that delay may arise if a "wrong" order is chosen. However, the "wrong" order does not arise in the equilibria because the buyer, by choosing the smaller seller to bargain with first, can avoid the delay and improve her payoff. Note that the delay is ensured by (12), without which there may not be delay when the wrong order is chosen, and there may be multiple equilibrium outcomes with different bargaining orders, as will be shown in Section 3.2.

3.2 Sellers of Similar Sizes

If the sellers are of similar sizes, (12) may be violated, which is equivalent to

$$\delta(1 - p_2^1) - v_1 > 0 \tag{28}$$

Step III in the proof of Proposition 4 already shows that the equilibrium outcome described in the proposition remains an equilibrium outcome. However, there are other equilibrium outcomes with different bargaining orders. Two other equilibria are described in the following two propositions. First, if we exchange sellers 1 and 2 in Step III in the proof of Proposition 4, we obtain a different set of equilibria. Specifically, swapping seller 1 and seller 2 in (13)-(16), we obtain

$$
\begin{aligned}
p_1^{2\prime} &= v_1 + \frac{\delta}{1+\delta}[\delta(1 - p_2^1) - v_1] \\
q_1^{2\prime} &= \delta(1 - p_2^1) - \frac{\delta}{1+\delta}\left[\delta(1 - p_2^1) - v_1\right] \\
p_2^{2\prime} &= H_{2,3} + \delta^3 p_2^1 \\
q_2^{2\prime} &= \delta(1 - p_1^1) - \delta(\delta(1 - p_2^1) - p_1^{2\prime})
\end{aligned}
$$

Condition (23) in Step III implies that if the buyer deviates to bargain with seller 1 first, there is a two-period delay. Swapping sellers 1 and 2 in (23), we obtain

$$\delta(1 - p_1^1) - v_2 < \delta^2[\delta(1 - v_2) - p_1^{2\prime}] \tag{29}$$

Under this condition, the following proposition describe a set of equilibria, in which if the buyer deviates to bargain with seller 2 first, there is a two-period delay.

Proposition 5 *In the infinite-horizon two-seller game, if (28) and (29) hold, then for any $\left(q_{2B}^{2\prime}, p_{B2}^{2\prime}\right)$ such that $q_{2B}^{2\prime} > q_2^{2\prime}$ and $p_{B2}^{2\prime} < p_2^{2\prime}$, the strategies below constitute an equilibrium:*

i) seller 1 accepts a price no less than $p_1^{2\prime}$ and suggests a price of $q_1^{2\prime}$,

ii) seller 2 accepts a price no less than $p_2^{2\prime}$ and suggests a price of $q_{2B}^{2\prime}$,

iii) the buyer bargains with seller 1 before the first purchase; accepts a price no more than $q_1^{2\prime}$ from seller 1 and no more than $q_2^{2\prime}$ from seller 2; and suggests $p_1^{2\prime}$ to seller 1 and $p_{B2}^{2\prime}$ to seller 2.

The proof is the same as in Step III, so it is omitted here. In the equilibrium, seller 1 sells in period 1 and seller 2 sells in period 2, and the payoff is

$$\pi_B^{*\prime} = \frac{1}{1+\delta}\left[\frac{\delta}{1+\delta}(1-v_2) - v_1\right] \tag{30}$$

for the buyer, $\pi_1^{*\prime} = v_1 + \frac{\delta}{1+\delta}\left[\frac{\delta}{1+\delta}(1-v_2) - v_1\right]$ for seller 1, and $\pi_2^{*\prime} = v_2 + \delta\frac{\delta}{1+\delta}(1-v_2)$ for seller 2. Note that there is no delay in the above equilibria, but there may be other equilibria with delay, which are characterized below.

Proposition 6 *In the infinite-horizon two-seller game, if (23), (28) and (29) hold, the following strategies constitute an equilibrium:*

i) the buyer chooses to bargain with seller 1 in period 1;

ii) everyone follows the strategies in Step I in the proof of Proposition 4 in $\Gamma(B,1)$;

iii) if the buyer chooses to bargain with seller 2 first, then everyone follows the strategies in Proposition 5 in $\Gamma(B,2)$.

Proof. If (28) holds, so does (11). Then, according to Step III in the proof of Proposition 4, under conditions (11) and (23) the above strategies constitute an equilibrium in subgame $\Gamma(B,1)$. As a result, this subgame has two periods of delay, after which the buyer switches to bargain with seller 2, then sellers 2 and 1 sell in periods 3 and 4 respectively. Thus, the buyer's payoff in $\Gamma(B,1)$ is $\delta^2\pi_B^*$ with π_B^* described in (19).

According to Proposition 5, under conditions (28) and (29), the above strategies constitute an equilibrium in subgame in $\Gamma(B,2)$. As a result, this subgame has two periods of delay, after which the buyer switches to bargain with seller 1, then sellers 1 and 2 sell in periods 3 and 4 respectively. Thus, the buyer's payoff in $\Gamma(B,2)$ is $\delta^2\pi_B^{*\prime}$ with $\pi_B^{*\prime}$ described in (30).

Hence, if the buyer bargains with seller 1 in period 1, the resulting subgame is $\Gamma(B,1)$, in which her payoff is $\delta^2\pi_B^*$. In contrast, if the buyer chooses seller 2 in period 1, the resulting subgame is $\Gamma(B,2)$, in which her payoff is $\delta^2\pi_B^{*\prime}$. Notice that $\delta^2\pi_B^* > \delta^2\pi_B^{*\prime}$, so the buyer prefers to bargain with seller 1 in period 1. ∎

Propositions 5 and 6 imply different bargaining orders: In Proposition 5, the buyer always bargains with seller 1, while in Proposition 6, the buyer first bargains with seller 1 then switches to seller 2. In the equilibria described in Proposition 6, there is no agreement in the first two periods and sellers 2 and 1 sell in periods 3 and 4 respectively. Thus, even though the bargaining game is of complete information, there may be delay in agreement. Several papers demonstrate delay in complete-information bargaining game. The reason discussed above is similar to Cai (2000). However, the reasons in Haller and Holden (1990) and Fernandez and Glazer (1991) are different. They consider two-person alternating bargaining games between a labor union and a firm, and the union can choose between production and strike when an offer is rejected. In an equilibrium with delay, the firm would rather wait several periods to avoid the "bad" equilibrium in which the union goes on strike once disagreement occurs. In our paper, production (building the mall) is not allowed while the bargaining is ongoing. Harvests are different to production in that the sellers receive them with certainty during the bargaining process.

Using the equilibrium payoffs in Propositions 4-6 as punishments, we can construct many other equilibria. Moreover, there is a continuum of equilibria without delays.[25] However, it is difficult to find the full characterization of equilibria or equilibrium payoffs even for the two-seller game. To see the potential difficulty, notice that in order to find the full characterization, we need to find the minimum and maximum for each player's payoffs as in the three-person alternating bargaining game.[26] Both the selling prices and the length of delay could affect the bounds. For example, seller 2's minimum payoff could be reached through a low selling price with a shorter delay or by a higher selling price but with a longer delay. Moreover, the two factors interact with each other and make the problem even more challenging. Since the maximum length of delay is very likely to be increasing in δ, the difficulty remains even when δ approaches 1. In an N-seller game, the above difficulty is amplified by the much larger number of possible selling orders.

4   Discussion and Applications

Inside vs. Outside Options   Suppose instead of receiving a harvest, each seller i at the end of each period has an option to exit the game by selling his land in an outside market for a price of v_i. In contrast to the inside options studied above, v_i is referred as to the outside option of seller i. It is well-known that, whether an alternating bilateral bargaining game has inside options or outside options, the players split the surplus in the same way.[27] Analogously, all of our results, Propositions 3-6 and Lemmas 1-3, hold if each seller i's outside option is v_i. The modification of the proofs in our game is a straightforward generalization of that in the

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[25] A similar analysis is used in the three-person alternating bargaining game. See, for instance, Herrero (1984) and Osborne and Rubinstein (1990). The equilibria with delay also demonstrate different bargaining orders as in Propositions 5 and 6, though they are not presented for the consideration of space.

[26] See, for instance, Herrero (1984), or Section 3.13 (pp. 63-65) of Osborne and Rubinstein (1990).

[27] For example, Section 3.9.1 of Osborne and Rubinstein (1990) shows that the player who proposes first receives $1/(1 + \delta)$ fraction of the total surplus, and the other receives $\delta/(1 + \delta)$ of it.

bilateral bargaining game, so it is omitted here.

Coordination Among Sellers   If the sellers can merge into a single agent who bargains on behalf of them, is it profitable to do so? The answer is no. To see this, suppose $\delta \to 1$ and suppose two sellers are represented by an agent. Then, the agent bargains as if he is in the one-seller game with size $v_1 + v_2$, and the resulting surplus is $(1 - v_1 - v_2)/2$ due to Lemma 1, which is lower than $[(1 - v_1)3/2 - v_2]/2$, the sum of the sellers' surpluses described in (17) and (18) if they bargain separately. The reason is that the buyer needs agreement from each seller, therefore each seller has "veto" power. If the sellers merge, there are less players with "veto" power, then their bargaining power is also reduced. Moreover, it may be beneficial for a seller to split his land and have several agents represent the different pieces.

Heterogeneous Discounting   Although we focus on a common discount factor above, our analysis applies to heterogeneous discount factors. To see this, suppose that, in the two-seller game, the discount factors are δ_1, δ_2, and δ_B for sellers 1, 2 and the buyer. Then, (8) becomes $p_i^1 = v_i + r_i (1 - v_i)$ where $r_i = (1 - \delta_B) \delta_i / (1 - \delta_i \delta_B)$, and equations (21) and (22) become $p_2^2 = v_2 (1 - \delta_2) + \delta_2 q_2^2$ and $\delta_B (1 - p_1^1) - q_2^2 = \delta_B (\delta_B (1 - p_1^1) - p_2^2)$, which imply $p_2^2 = v_2 + r_2 (\delta_B (1 - p_1^1) - v_2)$. Then, if the buyer purchases from seller 2 in period 1 then from seller 1 in period 2, the equilibrium prices are p_1^1 for seller 1 and p_2^2 for seller 2, and similar to (19), the buyer's payoff is $\pi_B = \delta_B (1 - p_1^1) - p_2^2 = (1 - r_2)(\delta_B (1 - r_1)(1 - v_1) - v_2)$. Similarly, if the buyer purchases from seller 1 in period 1 then from seller 2 in period 2, her payoff would be $\pi_B' = (1 - r_1)(\delta_B (1 - r_2)(1 - v_2) - v_1)$. Therefore, by the same proof for Proposition 4, we can generalize it as follows: If $\delta_B (1 - r_1)(1 - v_1) - v_2 > 0$ and $\delta_B (1 - r_2)(1 - v_2) - v_1 < 0$, which are analogues of (11) and (12), there is a unique equilibrium outcome, in which the buyer purchases from seller 2 in period 1 and from seller 1 in period 2. The other results can be generalized in the same way.

Order of Offers   So far we assumes that the buyer makes the first offer in each round of bargaining, however, our result is not hinged upon this assumption. For instance, suppose $\delta \to 1$ and the sellers make the first offer in each round of bargaining. Then, we can verify that the equilibrium prices and payoff are the same. What is important to our results is the inside/outside options, without which there could be multiple equilibria with different bargaining orders (see, for instance, Cai 2003 and Li 2010).

Beyond Perfect Complementarity   Our analysis apply to some scenarios in which the lots are not perfect complements. For instance, consider a four-seller game in which sellers $1, 2, 3, 4$ have values $0 < v_4 < v_3 < v_2 < v_1$. The buyer needs the land from sellers 1, 2, 3 or sellers 1, 2, 4 to build the mall, so the sellers are not perfect complements.[28] Assume that there is a

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[28]For example, see "A Holdout Against Developers Leaves a Legacy," *Seattle Journal*, December 27, 2008 for a report on a boutique supermarket that was altered from its original design and built around a house whose owner refused to sell.

finite horizon of 3 periods. By backward induction, we can verify that the prices must equal to the valuations because the sellers do not have time to make counter-offers.[29] Then, the buyer's payoff is $\pi_B = \delta(\delta(\delta(1-v_i)-v_j)-v_k)$, where k, j and i are the first, second and third sellers. If the mall is built, it must be that $\{i,j,k\} = \{1,2,3\}$ or $\{1,2,4\}$. Comparing the two possibilities, we can show that the buyer's payoff is maximized at $\pi_B = \delta(\delta(\delta(1-v_1)-v_2)-v_3)$. Hence, the buyer also bargains with sellers in the order of increasing size. It is an interesting question to consider the equilibrium bargaining order with more sellers and longer horizons, but the setup would be quite different from the current one. If the lots are perfect substitutes, the buyer may prefer to bargain with the larger seller first. See, for instance, Krasteva and Yildirim (2012).

Cash-Offer vs. Contingent Contracts   This paper considers cash-offer contracts, which are prevalent in the real estate business.[30] In addition, it would be difficult to settle with a union on strike by a contract contingent on future events.[31] If the buyer can use contingent contracts, with which the payments are not made until all the sellers have agreed, there are still equilibria with the bargaining order of increasing size. However, other bargaining orders may also arise in equilibria even under the assumptions in Proposition 3.[32]

Simultaneous vs. Sequential Offers   In many situations, it is difficult or impossible for the buyer to ensure the sellers receive the offers simultaneously, and probably more difficult for sellers to reply simultaneously. If these simultaneous actions are possible, there is an equilibrium in which all the sellers agree in the first period because this game is of complete information. Therefore, we cannot compare bargaining orders in such a setup.

Applications and Other Extensions   Besides land purchasing and the two other examples in the introduction, the model is also applicable to voting scenarios. For example, when a country wants to join a trade organization, it has to receive permission from all the respective existing members. The members have different attitudes toward the entry, and the member that prefers the entry least corresponds to the seller with largest size in our model. As a result, the applicant should start with the member who favors the entry most. Moreover, our model can be modified to study the voting situations where winning requires not only a minimum number of votes but also all the votes from voters with veto rights. Another extension is to allow some players to hide information such as the sellers' sizes, past offers or deal prices.[33] It would be interesting to examine how bargaining orders affect the players' incentive to reveal their private information.

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[29]Each player is assumed to participate if his/her surplus is nonnegative.

[30]See "Nail House in Chongqing Demolished," *China Daily,* April 3, 2007.

[31]In a strike bargaining started in 2005, the Northwest Airlines settled with the Aircraft Mechanics Fraternal Association (AMFA), which represented aircraft mechanics, janitors, and aircraft cleaners, in November 2006, then settled with the flight attendant union half a year later in May 2007. The agreements with AMFA were not conditional on the later negotiation. See Pongrace (2007) for more details.

[32]See, for instance, Suh and Wen (2009) and Li (2010).

[33]Noe and Wang (2004) examine a bargaining game in which the negotiation history could be kept secret, and Chowdhury and Sengupta (2012) study how transparency of bargaining protocol affects outcome's efficiency.

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Appendix

Proof of Proposition 2. We prove it for $N = 2$ and the proof for $N > 2$ is similar. We use Proposition 1 and its proof as a building block to prove Proposition 2. Specifically, we use the following properties established in the online appendix:[34]

i) Suppose the buyer and seller 1 participate the one-seller game with horizon $T \geq 1$, then the mall is built, and the buyer's equilibrium payoff converges to $\pi_{B,\infty}^1 = (1 - v_1)/(1 + \delta)$ as $T \to \infty$.

ii) In the two-seller game, if the mall is built for horizon T, it is also built for horizon $T + 2$, and $\pi_{B,T+2}^2 = (\delta \pi_{B,T+1}^1 - v_2) - \delta(\delta \pi_{B,T}^1 - v_2) + \delta^2 \pi_{B,T}^2$.

iii) In the two-seller game, the mall is built for an even horizon $T \geq 4$ if and only if $\delta \pi_{B,T-2}^1 - v_2 > \delta(\delta \pi_{B,T-3}^1 - v_2)$. Moreover, if the mall is built for the even horizon T, $\pi_{B,T+2}^2 < \pi_{B,T}^2$.

iv) In the two-seller game, the mall is built for an odd horizon $T \geq 3$ if and only if $\delta \pi_{B,T-1}^1 - v_2 > \delta(\delta \pi_{B,T-2}^1 - v_2)$. Moreover, if the mall is built for the odd horizon T, $\pi_{B,T+2}^2 > \pi_{B,T}^2$.

Property i) implies that as $T \to \infty$, the buyer and seller 1 split the total surplus $1 - v_1$ as in the Rubinstein bargaining game, i.e., $1/(1 + \delta)$ of the surplus goes to the buyer and $\delta/(1 + \delta)$ of it to the seller. Property ii) provides a recursive formula of the buyer's equilibrium payoffs. According to properties iii) and iv), the buyer's equilibrium payoffs for even (odd) horizons are a decreasing (increasing) sequence.

Consider Proposition 2 for even horizons first. Notice that ii) implies that the condition for the mall to be built becomes weaker for longer even horizons. In addition, if an even $T \to \infty$, the condition in iii) becomes $\delta \pi_{B,\infty}^1 - v_2 > \delta(\delta \pi_{B,\infty}^1 - v_2)$, which is equivalent to (3) for $N = 2$. Hence, for all even horizons, (3) is the weakest condition for the mall to be built. In other words, if (3) does not hold, the mall is not built for any even horizon.

Recall that the condition for the mall to be built is weaker with longer even horizons and that the limiting condition is (3) if an even $T \to \infty$, so there is an even T_e^2 such that the mall is built for even horizons $T > T_e^2$, where the superscript represents the number of sellers in the game. Moreover, iii) implies that the buyer's equilibrium payoffs $\{\pi_{B,T}^2\}$ for even horizons $T > T_e^2$ is a decreasing sequence with a lower bound at 0, so it converges. Moreover, ii) implies $\lim_{2t \to \infty} \pi_{B,2t}^2 = (\delta \pi_{B,\infty}^1 - v_2) - \delta(\delta \pi_{B,\infty}^1 - v_2) + \delta^2 \lim_{2t \to \infty} \pi_{B,2t}^2$, so $\lim_{2t \to \infty} \pi_{B,2t}^2 = \frac{1}{1+\delta}(\delta \pi_{B,\infty}^1 - v_2)$. Because seller 1 sells in period 2 according to Proposition 1, the payoff for the buyer is $\pi_{B,2t}^2 = \delta \pi_{B,2t-1}^1 - p_{2,2t}^2$. Therefore, $p_{2,2t}^2$ also converges and $\lim_{2t \to \infty} p_{2,2t}^2 = v_2 + \frac{\delta}{1+\delta}(\delta \pi_{B,\infty}^1 - v_2)$.

Consider odd horizons. Similar to the even horizons, using properties ii) and iv), we can show that if (3) does not hold, the mall is not built for any odd horizon. Moreover, there is also an odd T_o^2 such that the mall is built for odd horizons $T > T_o^2$. Property iv) implies that the buyer's equilibrium payoffs $\{\pi_{B,T}^2\}$ for odd horizons $T > T_o^2$ is an increasing sequence with an

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[34]In the online appendix, see Claim 1 for i), Claim 11 for ii), Claims 6 and 10 for the conditions for the mall to be built in iii) and iv), and Claim 11 for the payoff comparisons in iii) and iv).

upper bound at 1, so it converges. Then, similar to the even horizons, we can use ii) to show that $\lim_{2t+1\to\infty}\pi^2_{B,2t+1}=\frac{1}{1+\delta}(\delta\pi^1_{B,\infty}-v_2)$ and $\lim_{2t+1\to\infty}p^2_{2,2t+1}=\frac{\delta}{1+\delta}(\delta\pi^1_{B,\infty}-v_2)$.

Hence, combining the results for even and odd horizons, we have $\lim_{T\to\infty}\pi^2_{B,T}=\frac{1}{1+\delta}(\delta\pi^1_{B,\infty}-v_2)$ and $\lim_{T\to\infty}p^2_{2,T}=v_2+\frac{\delta}{1+\delta}(\delta\pi^1_{B,\infty}-v_2)$. Similarly, we can prove $\lim_{T\to\infty}\pi^N_{B,T}=\frac{1}{1+\delta}(\delta\pi^{N-1}_{B,\infty}-v_N)$ and $\lim_{T\to\infty}p^N_{N,T}=v_N+\frac{\delta}{1+\delta}(\delta\pi^{N-1}_{B,\infty}-v_N)$. ∎

Proof of Corollary 1. If $N=2$, condition (3) becomes $\frac{\delta}{1+\delta}(1-v_1)-v_2>0$. As in property i), the buyer's equilibrium payoff in the one-seller game satisfies $\lim_{T\to\infty}\pi^1_{B,T}=\frac{1}{1+\delta}(1-v_1)$. Then, Proposition 2 implies that $\lim_{T\to\infty}\pi^2_{B,T}=\frac{1}{1+\delta}(\delta\lim_{T\to\infty}\pi^1_{B,T}-v_2)=\frac{1}{1+\delta}(\frac{\delta}{1+\delta}(1-v_1)-v_2)$ and $\lim_{T\to\infty}p^2_{2,T}=v_2+\frac{\delta}{1+\delta}(\delta\lim_{T\to\infty}\pi^1_{B,T}-v_2)=v_2+\frac{\delta}{1+\delta}(\frac{\delta}{1+\delta}(1-v_1)-v_2)$. ∎

The remainder of the appendix studies the bargaining game with an infinite horizon. The lemma below is used in Step II in the proof of Proposition 4.

Lemma 2 *In the infinite-horizon two-seller game, if (11) and (12) hold, perpetual disagreement does not arise in the subgame given every player's participation.*

Proof. Suppose otherwise that there is an equilibrium with perpetual disagreement, then the buyer's payoff is 0. We show below that the buyer can deviate and obtain a positive payoff by bargaining with seller 2 first.

First, the payoff of seller 2 in the subgame $\Gamma(2,B)$ is at most $\delta(1-p^1_1)$. To see this, notice that in $\Gamma(2,B)$, seller 2 offers to the buyer in period 1. If there is perpetual disagreement in $\Gamma(2,B)$, seller 2's payoff is $\pi_2=v_2$, which is lower than $\delta(1-p^1_1)$ due to the expression of p^1_1 in (8) and condition (11). Thus, the claim is true in this case.

If there is an agreement in $\Gamma(2,B)$, the first seller must be seller 2. Otherwise, the buyer's payoff is $\delta(1-p^1_2)-p^2_1\leq\frac{\delta}{1+\delta}(1-v_2)-v_1<0$, where the first inequality is from (8) and $p^2_1\geq v_1$ and the second from (12). Therefore, if subgame $\Gamma(2,B)$ has an agreement, seller 2 sells first. Suppose he sells in period t_2 in the subgame, then the buyer's payoff is $\delta^{t_2-1}(\delta(1-p^1_1)-p^2_2)\geq 0$, which implies $p^2_2\leq\delta(1-p^1_1)$. Therefore, seller 2's payoff satisfies $\pi_2=H_{2,t_2-1}+\delta^{t_2-1}p^2_2=v_2+\delta^{t_2-1}(p^2_2-v_2)\leq v_2+\delta^{t_2-1}(\delta(1-p^1_1)-v_2)\leq v_2+\delta(1-p^1_1)-v_2=\delta(1-p^1_1)$, where the first inequality is from $p^2_2\leq\delta(1-p^1_1)$ and the second from $t_2\geq 1$. Thus, the claim is also true in this case.

Second, suppose there is an equilibrium with perpetual disagreement, we construct a deviation for the buyer below. Suppose the buyer deviates in period 1 by offering seller 2 a price of $p^2_2=v_2+\delta[\delta(1-p^1_1)-v_2]+\varepsilon$ with $\varepsilon>0$. Then, seller 2 accepts it otherwise he receives at most $v_2+\delta[\delta(1-p^1_1)-v_2]$, which is the sum of his harvest $v_2(1-\delta)$ in this period and the payoff upper bound $\delta(1-p^1_1)$ in the next period derived in the first step. Substituting p^1_1 described in (8) and p^2_2 into it, we obtain the buyer's payoff $\pi_B=\delta(1-p^1_1)-p^2_2=(1-\delta)[\frac{\delta}{1+\delta}(1-v_1)-v_2]-\varepsilon$. Since the first term is positive due to (11), there is a small enough ε such that $\pi_B>0$. Hence, we find a profitable deviation for the buyer, so perpetual disagreement cannot arise in an equilibrium. ∎

In order to prove Proposition 3, we first show Proposition 4, which is the two-seller version of Proposition 3, then we generalize the analysis to the N-seller game. Since we have shown Proposition 4, let us now consider the N-seller game.

We first generalize the notation in the two-seller game to the N-seller game. For any $i \in \{2, ..., N\}$, suppose the buyer has a unique equilibrium payoff in the game with sellers $1, ..., i-1$, which will be proved to be true under the assumptions in Proposition 3, and denote the payoff as π_B^{i-1*}. Recall that the superscript indicates the number of sellers. Then, define

$$p_i^i = v_i + \frac{\delta}{1+\delta}[\delta\pi_B^{i-1*} - v_i] \tag{31}$$

$$q_i^i = \delta\pi_B^{i-1*} - \frac{\delta}{1+\delta}[\delta\pi_B^{i-1*} - v_i] \tag{32}$$

which reduce to (13) and (14) if $i = 2$. If seller i sells in period 1 at price p_i^i, which is indeed the case under the assumptions in Lemma 3, the buyer's payoff is $\pi_B^{i*} = \delta\pi_B^{i-1*} - p_i^i = \frac{1}{1+\delta}[\delta\pi_B^{i-1*} - v_1]$, where the second equality is from (31). Recall that $\pi_B^{1*} = 1 - p_1^1 = \frac{1}{1+\delta}[1 - v_1]$, so the above recursive formula implies

$$\pi_B^{i*} = \frac{1}{1+\delta}\left(\left(\frac{\delta}{\delta+1}\right)^{i-1} - \sum_{j=1}^{i}\left(\frac{\delta}{1+\delta}\right)^{i-j}v_j\right) \equiv \Pi_B^i(\mathbf{v}^i)$$

where $\mathbf{v}^i = (v_1, ..., v_i)$. Note that $\Pi_B^i(\mathbf{v}^i)$ is an affine function of $\mathbf{v}^i$, and for $j = 1, ..., i-1$, the coefficient of v_j is negative and has a smaller absolute value than that of v_{j+1}. For $i = 1, ..., N-1$, the two equations below generalize (15) and (16):

$$p_i^N = H_{i,2+N-i} + \delta^{2+N-i}p_i^i \tag{33}$$

$$q_{Bi}^N = \delta\Pi_B^{N-1}(\mathbf{v}_{-i}) - \delta[\delta\pi_B^{N-1*} - p_N^N] \tag{34}$$

where $\mathbf{v}_{-i}$ is the vector $(v_1, ..., v_N)$ with v_i removed. With p_i^i, q_i^i, p_i^N and q_{Bi}^N defined as in (31)-(34), we characterize a set of equilibria in the following lemma, which is an analogue of Step I in the proof of Proposition 4.

Lemma 3 *If (7) for $n = 2, ..., N$ and (3) are satisfied, given any (q_i^N, p_{Bi}^N) such that $q_i^N > q_{Bi}^N$ and $p_{Bi}^N < p_i^N$, the following strategies constitute an equilibrium in the infinite-horizon N-seller game:*

i) seller $i \in \{1, ..., N\}$ suggests a price of q_i^N and accepts a price no less than p_i^N,

ii) the buyer bargains with seller N before the first agreement; suggests a price of p_N^N to seller N and a price p_{Bi}^N to seller $i = 1, \cdots, N-1$; and accepts a price no more than q_N^N from seller N and a price no more than q_{Bi}^N from seller $i = 1, \cdots, N-1$.

Proof. Induction on the number of sellers is used. If $N = 2$, Lemma 3 becomes the claim in Step I in the proof of Proposition 4, so it is true. Suppose the lemma is true for $N = n \geq 2$, and we want to show it is also true for $N = n + 1$. First, we can verify that (3) for $N = n + 1$

implies that (3) for $N = 2, ..., n$. By the same backward induction analysis in Step I in the proof of Proposition 4, we can verify that no player deviates from the proposed strategies in an $(n+1)$-seller game. Therefore, we do not repeat the proof here and only discuss interpretations below.

Analogous to (15) and (16), equation (33) ensures that seller $i < N$ is indifferent between accepting and rejecting the buyer's offer p_i^N, and (34) ensures that the buyer is indifferent between accepting and rejecting seller i's offer q_{Bi}^N. The sellers sell in the order of increasing size in the first N periods. If seller $N-1$ and seller N exchange their selling periods, the buyer's payoff is $\Pi_B^N(v_1, ..., v_{N-2}, v_N, v_{N-1})$, and condition (7) for $n = N$ is equivalent to $\Pi_B^N(v_1, ..., v_{N-2}, v_N, v_{N-1}) < 0$. Recall that v_i's coefficient is negative and has a smaller absolute value than that of v_{i+1}'s in Π_B^N for all $i < N$, condition (7) also implies a negative payoff for the buyer if any seller other than N sells first. In other words, this condition for $n = N$ ensures the smallest seller N sells first. Moreover, (7) holds for $n = 2, ..., N$, so the smallest remaining seller n has to be the first to sell in the subgame with sellers $1, ..., n$. ∎

Proof of Proposition 3. The proof is based on induction on the number of sellers. If $N = 2$, Proposition 3 reduces to Proposition 4, so it holds. Next, suppose that for $n = 2, ..., N-1$, Proposition 3 holds in the n-seller game, then we want to show the proposition in the N-seller game. Because the proof is similar to that of Proposition 4, we only sketch it below.

First, as in Step I in the proof of Proposition 4, Lemma 3 characterizes a set of equilibria. As in Step II of Proposition 4, we can verify that if (3) holds and (7) holds for $n = 1, ..., N$, seller N must sell first in the N-seller game, otherwise the buyer's payoff is negative. Then, as in the remainder of Step II, we can prove that the outcome associated with the strategies in Lemma 3 is the unique equilibrium outcome, which proves part a) of the proposition. Finally, by the identical argument in Step III of Proposition 4, we can prove parts b) and c) of Proposition 3. ∎